Towards revision of conventional theory and modelling of turbulence in boundary layer flows

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Numerical Weather and Climate Modeling:
Beginnings, Now, and Vision of the Future
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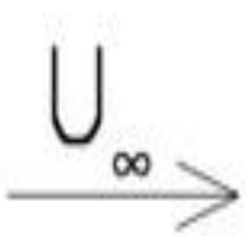
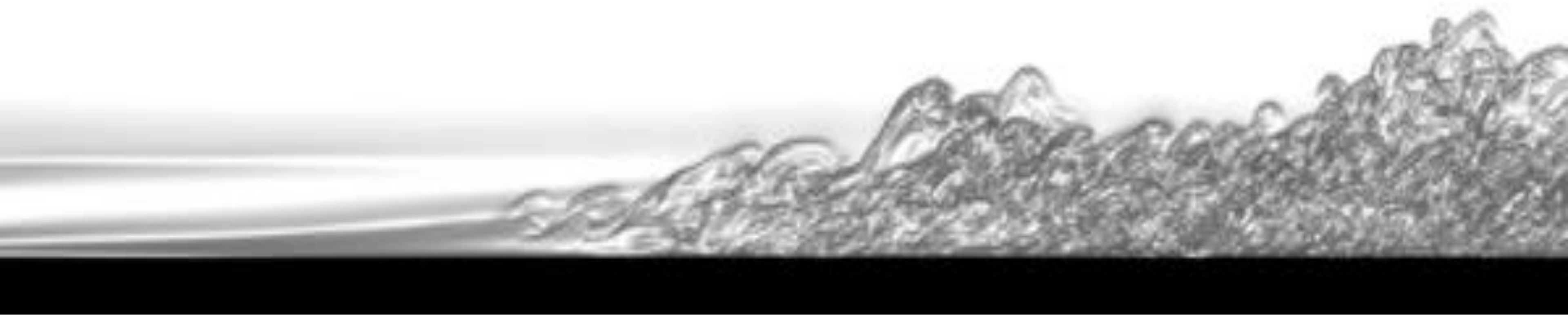
Conventional vision of <u>TURBULENCE</u>



- **<u>Two types</u> of motion: regular mean flow + chaotic turbulence charaterised by direct cascade**
- **Energetics fully defined by the Turbulent Kinetic Energy (TKE) budget equation**
- **Turbulent fluxes = gradients multiplied by exchange coefficients: eddy viscosity, conductivity, diffusivity**



<u>Chaos our of order</u> (Kolmogorov, 1941)

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Current and proposed revised paradigms

CURRENT PARADIGM of theory of turbulence (the *forward cascade* towards *dissipation and* the *downgradient fluxes*) is attributed to Kolmogorov (K-1941-1942); however he limited to shear-generated turbulence in neutrally stratified flows

His followers extended the paradigm without proof to both:

- **unstable stratification: buoyancy-generated plumes principally different from shear-generated eddies**
- **stable stratification: believed to "consuming" turbulent kinetic energy (TKE), but in fact converting TKE into turbulent potential energy (TPE)**

REVISED PARADIGM takes into account

- **self-organisation in unstable stratification: inverse cascade of TKE → its conversion into KE of self-organised motions**
- **self-control in stable stratification via countergradient heat flux**

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Stable and Neutral Planetary Boundary Layer (PBL) models overestimate mixing and height of the PBL

This results in essential errors in determining the most important near-surface parameters

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<u>Self-control</u> of turbulence in stable stratification via counter-gradient heat flux missed in K-1941, MO-1954

F_θ-budget reveals <u>downgradient</u> and <u>countergradient</u> terms comprising the vertical heat flux

$$F_\theta = C_1 t_T \beta \langle \theta^2 \rangle - C_2 t_T E_z \frac{\partial \Theta}{\partial z}$$

Key feedback assuring self-control (Z et al., 2007, 2013):

An **increase** in temperature gradient $\partial\Theta / \partial z$ **enhances**

(1) total (**<u>negative</u>**) fluxes of heat F_θ and buoyancy $F_b = \beta F_\theta$,

(2) hence, mean squared temperature $\langle \theta^2 \rangle = -C_3 t_T F_\theta \, \partial\Theta / \partial z$

(3) hence, countergradient <u>positive</u> contribution to heat flux $C_1 t_T \langle \theta^2 \rangle$

This compensates for the enhancing of negative heat flux and <u>prevents collapse of turbulence in super-critical stratification</u>

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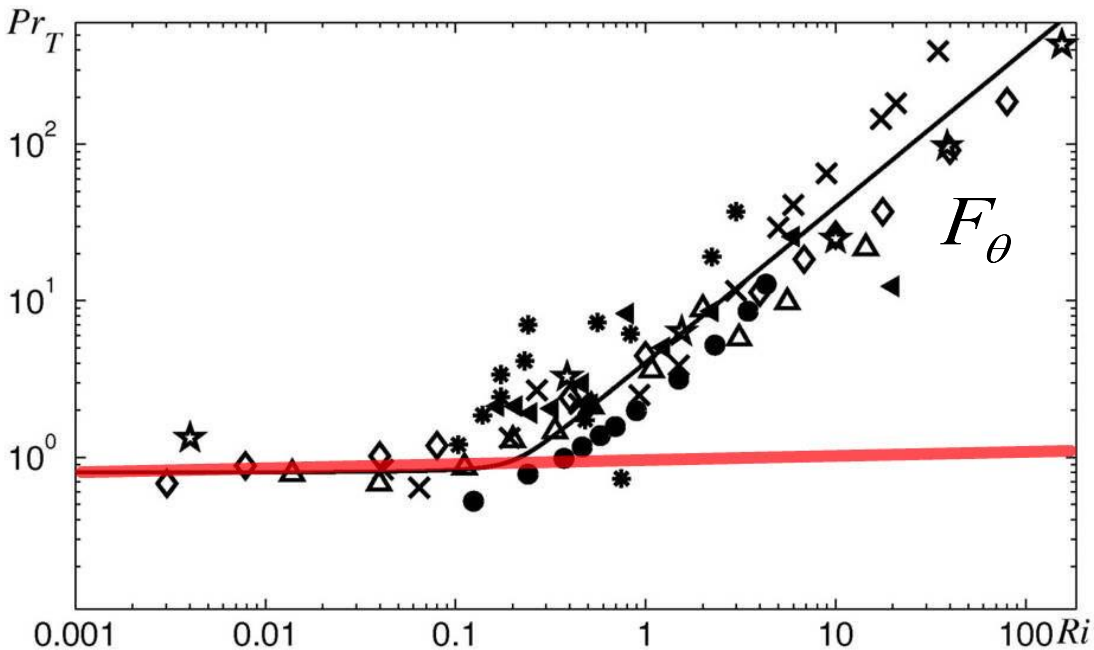
Prandtl no. Pr_T vs. Richardson no. Ri

K-1941, MO-1954 ignore self-control of heat flux, F_ϑ, and suggest the similar viscosity and conductivity: $Pr_T = K_M / K_H = constant$

This suggests erroneous turbulence cut off at $Ri > Ri_c = 0.25$



Black line: Pr_T after the EFB turbulence closure (Z et al., 2007-2018)

Red line: Pr_T prescribed by conventional theories (e.g. MO-1954)

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Stable stability: ***strong-mixing*** PBL turbulence and ***weak-conductivity*** turbulence aloft ($Ri > Ri_c$)

Shallow PBL is seen due to water haze (Bergen). Traditional theory does not distinguish between turbulence in **weakly** stable PBL and **supercritically** stable free flow. The problem is solved by EFB closure (Z et al., 2007-2018).

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EFB turbulence closure (Zilitinkevich *et al.*, 2013)

- Budget equations for basic second moments: E_K , E_P , τ_i (i = 1, 2) and F_z
- New prognostic equation for TKE dissipation rate ε_T
- Theory covers non-steady turbulence accounting for non-gradient and non-local transports
- Resolves supercritical turbulence and reveals two principal regimes:

  Mixing turbulence in boundary layer flows: $K_M \sim K_H$ at $Ri < Ri_c$

  Wave-like turbulence in free atmosphere (FA): $Pr_T = K_M / K_H \sim 4\, Ri$ at $Ri >> Ri_c$
- Calibration and testing needed

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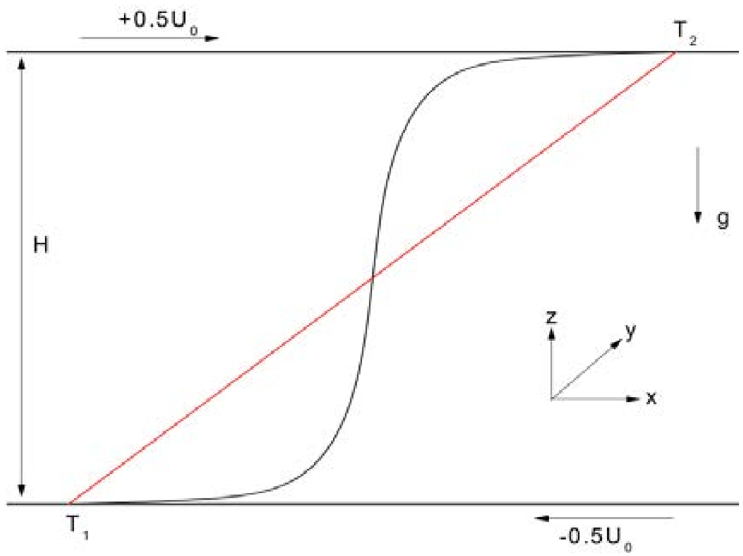
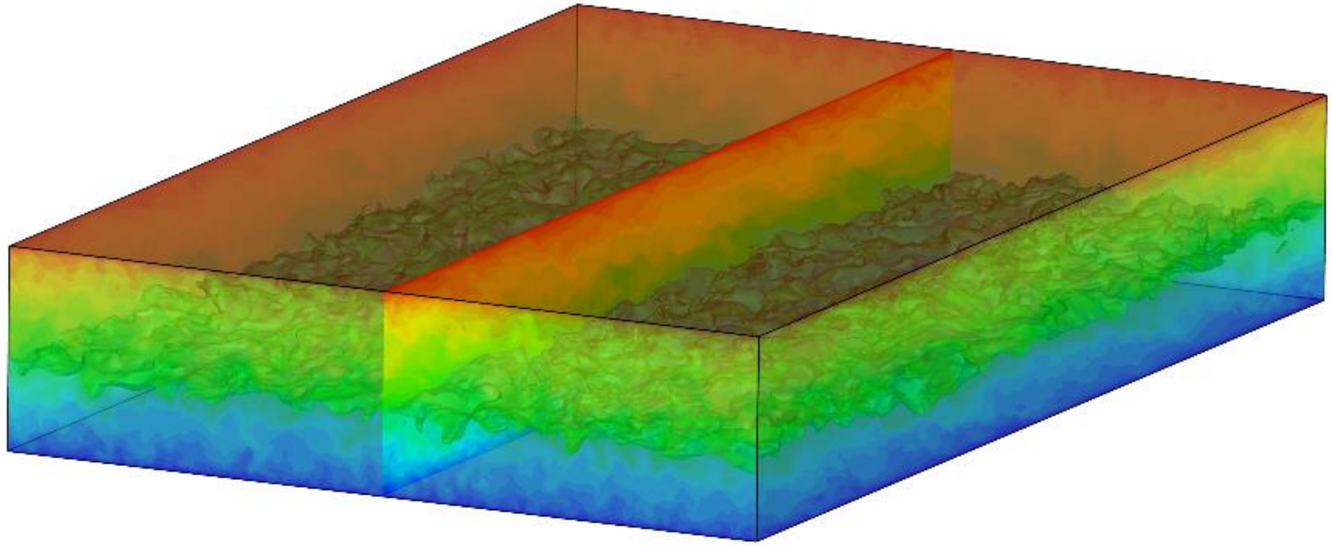
DNS: Stably stratified Couette flow

Couette flow – the flow between two parallel plates moving in opposite directions:

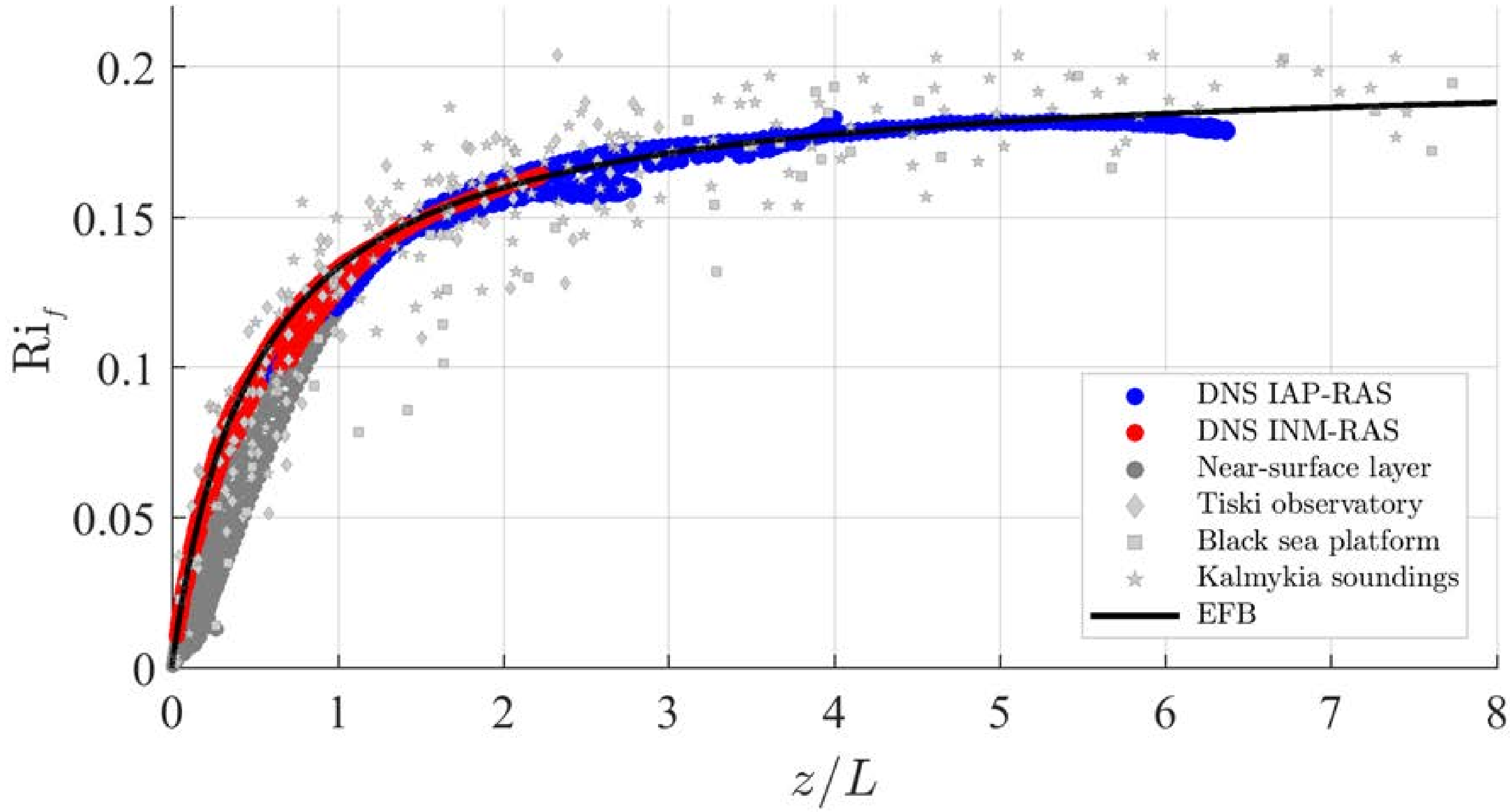
- Simple model of shear-driven flow
- Plane geometry, periodic BCs in horizontal directions
- Constant shear stress
- Statistically stationary flow
- Stable stratification





Maximal Flux Richardson number

$$\mathrm{Ri}_f \equiv \frac{\beta F_z}{\boldsymbol{\tau}\cdot \partial \mathbf{U}/\partial z} = \frac{kz/L}{1+kz/R_\infty L}$$

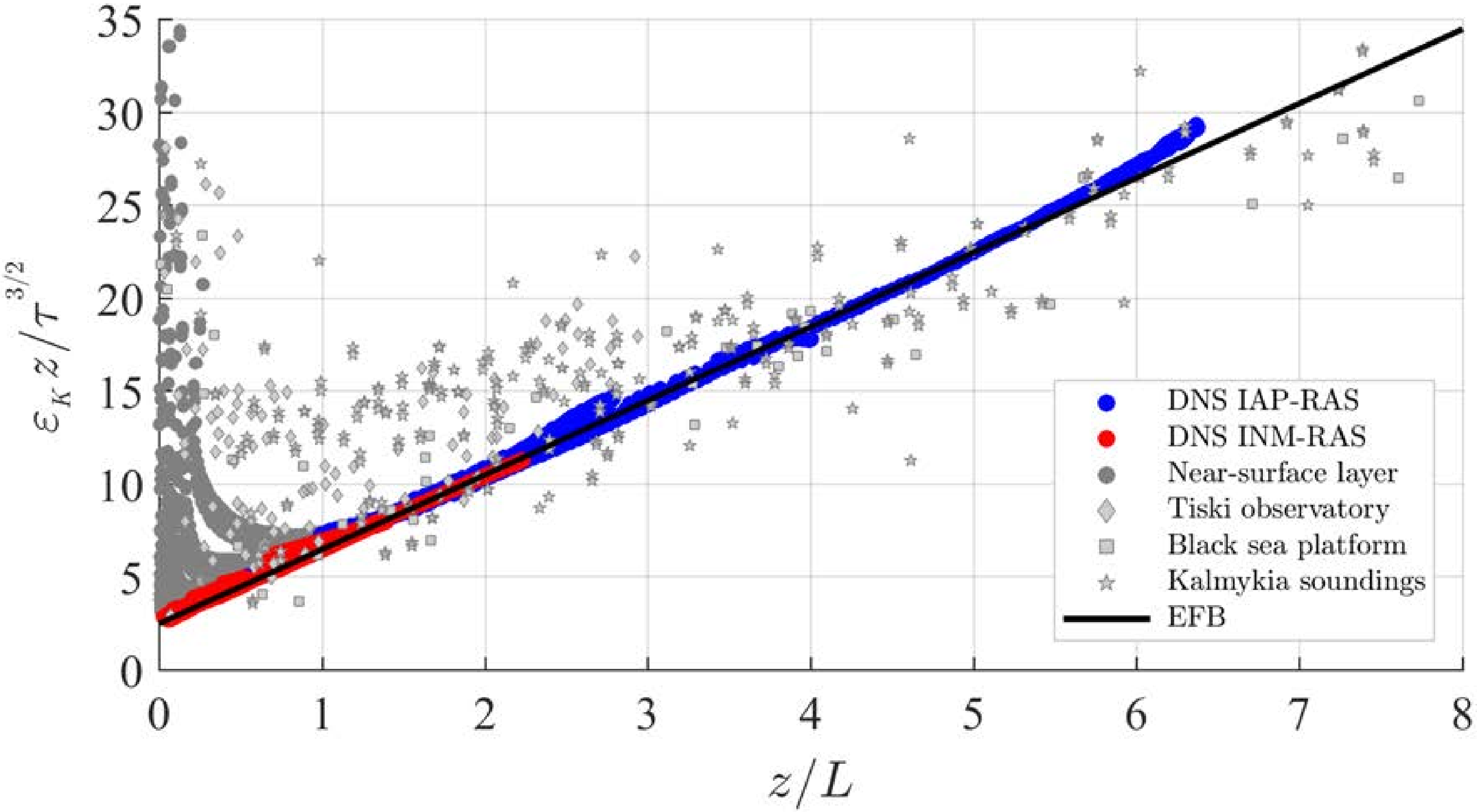


Flux Richardson number versus *z*/*L*, where *L* is Obukhov length scale $\left(L = \frac{\tau^{3/2}}{-\beta F_z}\right)$

Black solid line – best fit of EFB to DNS data

Steady-state TKE dissipation rate

$$\varepsilon_K = \frac{\tau^{3/2}}{kz}\left[1+\left(R_\infty^{-1}-1\right)kz/L\right]$$

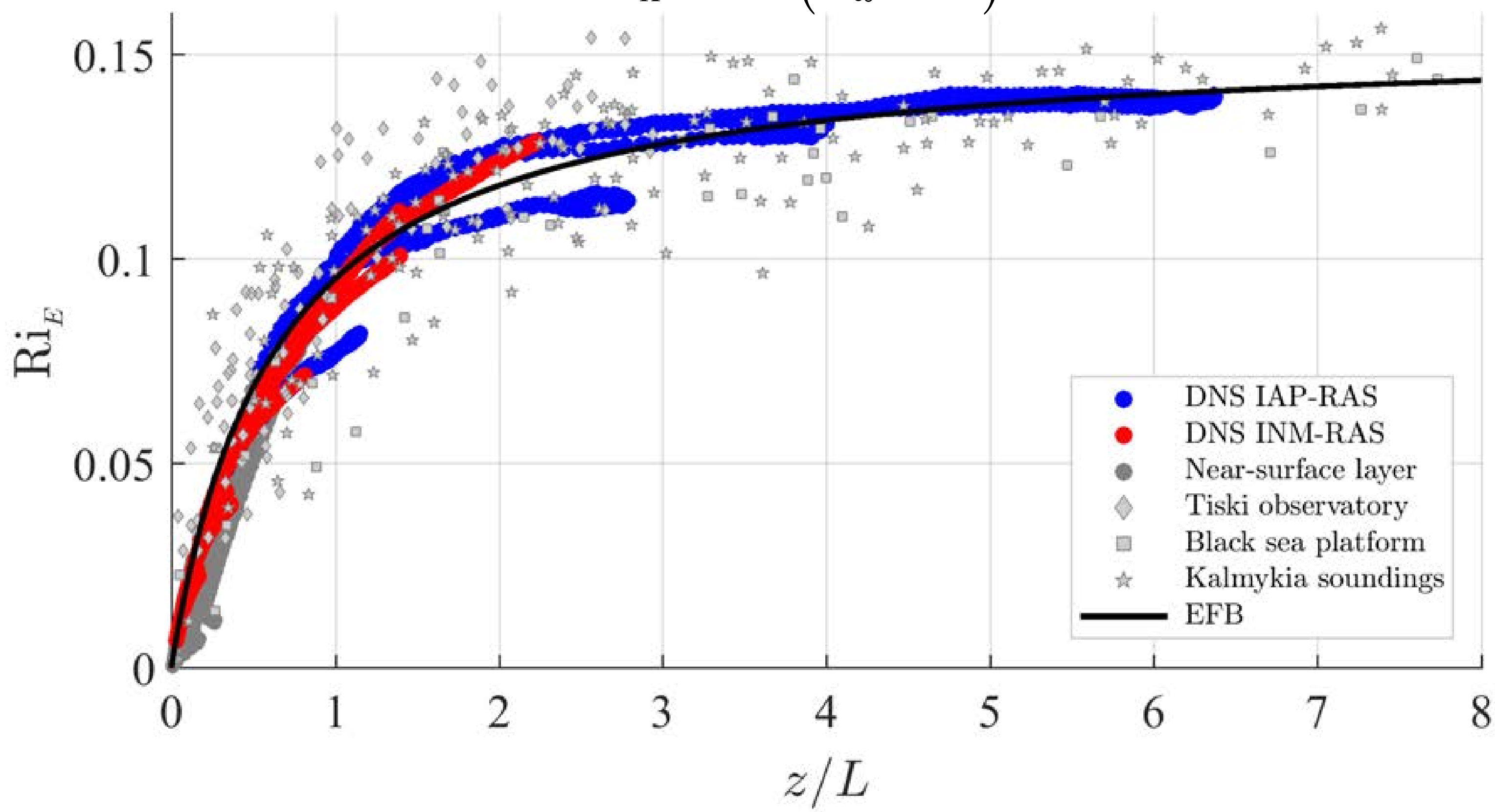


Dimensionless dissipation rate versus *z/L*

Theoretical curve (black solid line) is fully consistent with experimental data

Energy Richardson number for any heterogeneous and non-stationary flows

$$\mathrm{Ri}_E \equiv \frac{E_P}{E_K} = \frac{C_P kz/L}{1+\left(R_\infty^{-1}-1\right)kz/L}$$



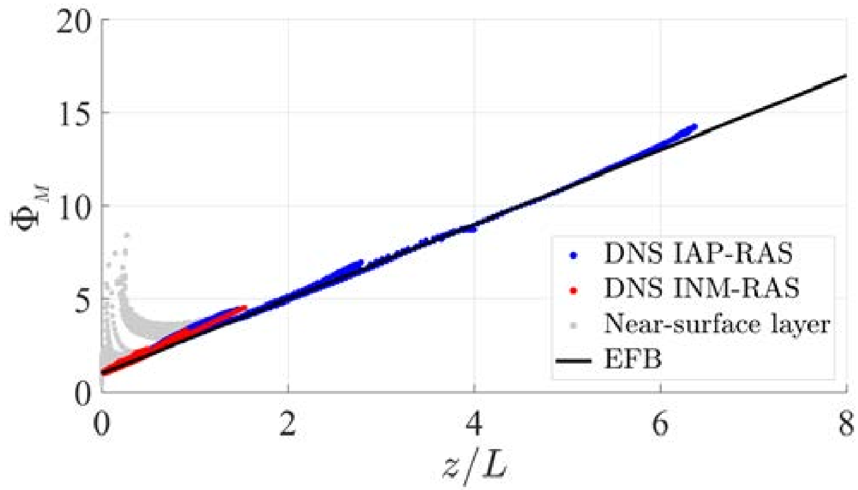
Energy Richardson number versus *z/L*

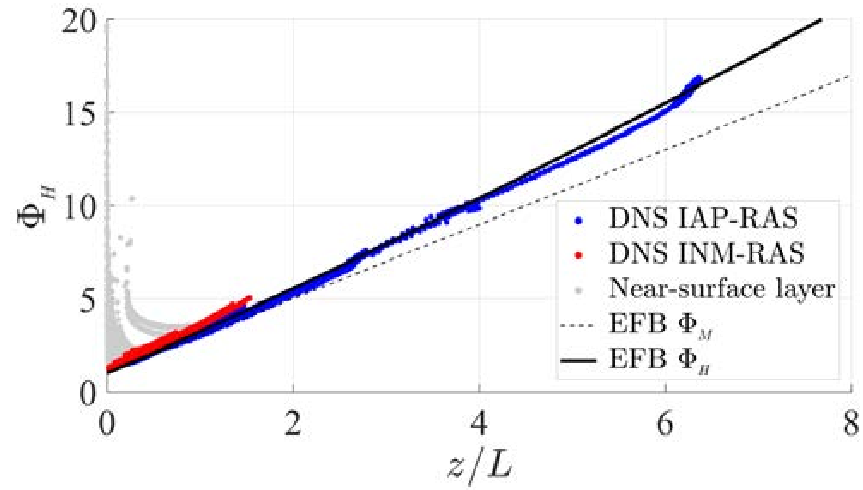
Black solid line – best fit of EFB to DNS data

Dimensionless velocity and potential temperature gradients as functions of *z/L*

$$\Phi_M \equiv \frac{kz}{u_*}\frac{du}{dz}$$

$$\Phi_H \equiv \frac{k_T z}{T_*}\frac{d\theta}{dz}$$





Dimensionless wind-velocity gradient Φ_M and Dimensionless potential temperature gradient Φ_H versus z/L

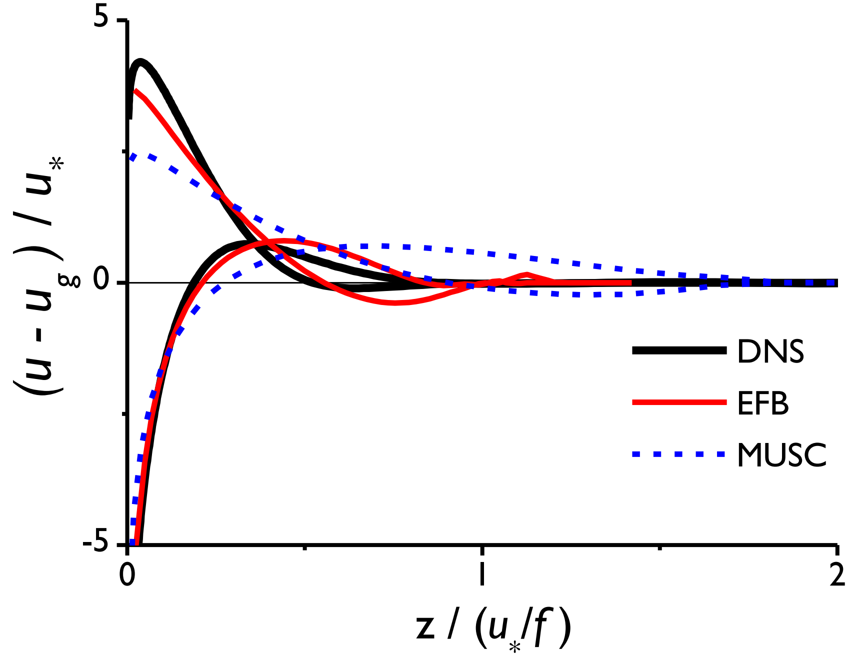
Φ_H increases faster then Φ_M assuring non-constant Pr_T

Truly neutral PBL (Ekman layer)

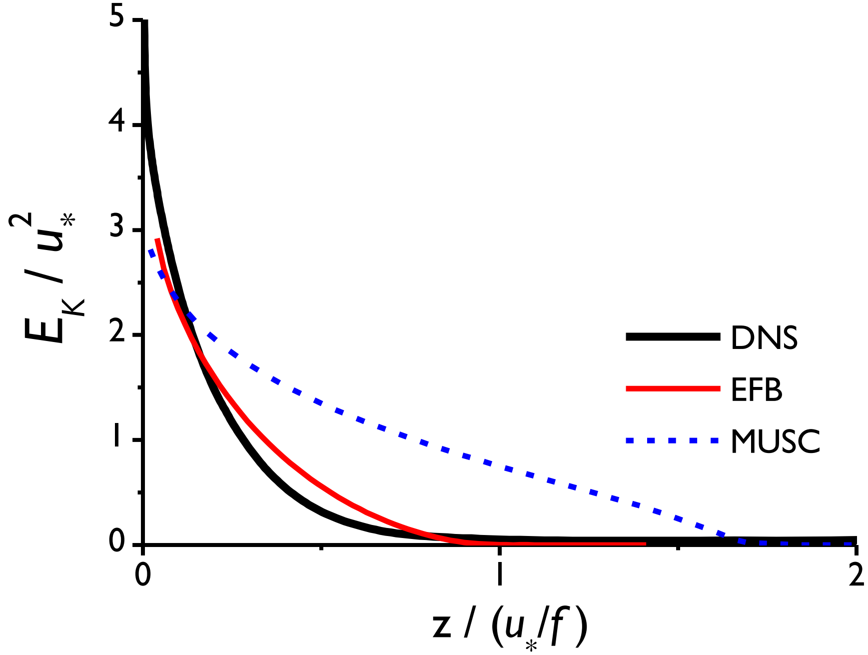
PBL with height-constant potential temperature formed by pressure gradient in rotating system

Very reliable DNS data from Spalart et. al. (2008)

Two RANS model runs: EFB and MUSC (HARMONIE/AROME weather prediction system)



Dimensionless profiles of wind velocity components



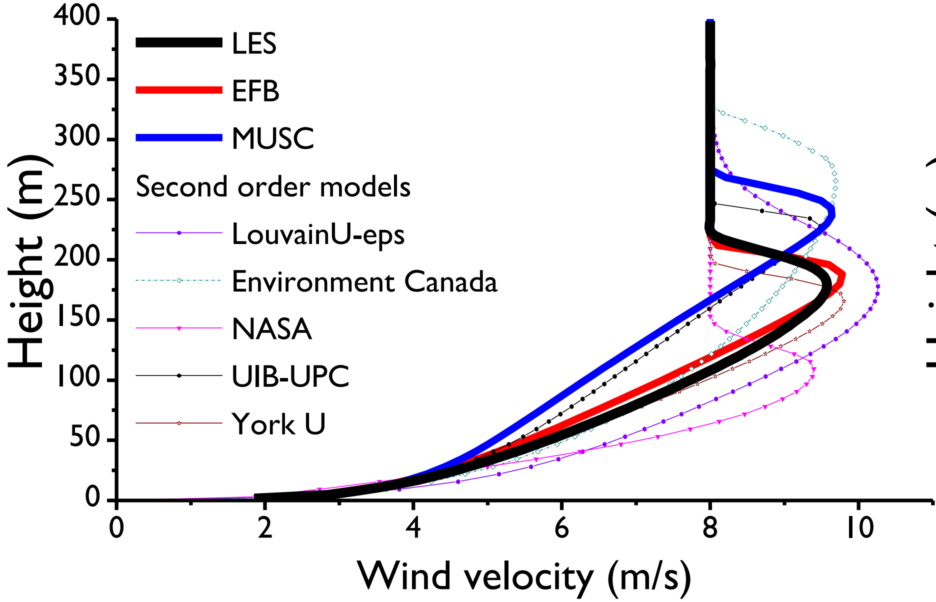
Dimensionless turbulent kinetic energy

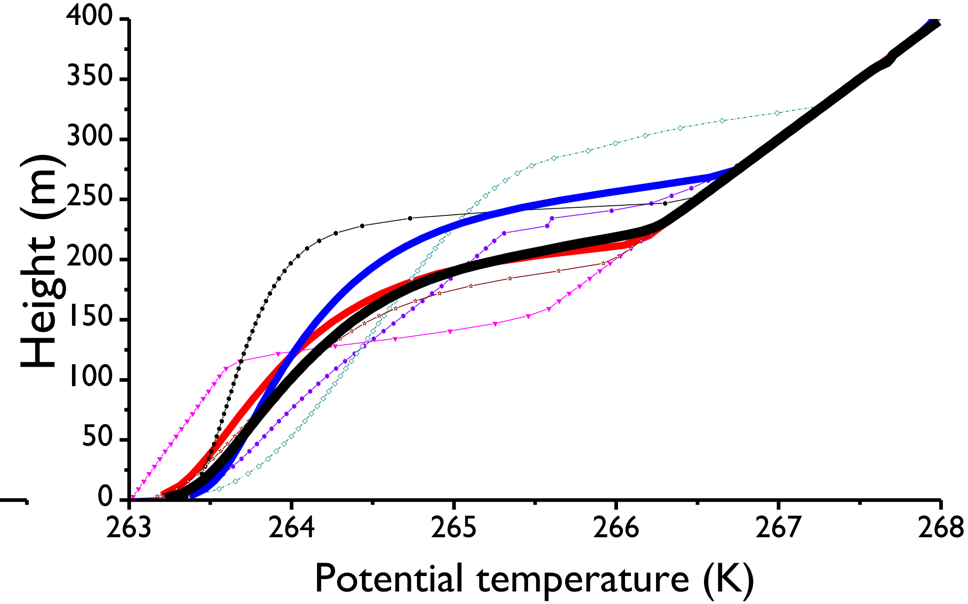
versus dimensionless height

EFB correctly models PBL height

Stably stratified idealized GABLS1 case

Initially 100 m deep vertically homogeneous layer evolves against stable stratification controlled by persistent cooling of the surface. *GABLS = GEWEX Atmospheric Boundary-Layer Study (Holtslag, 2003)*

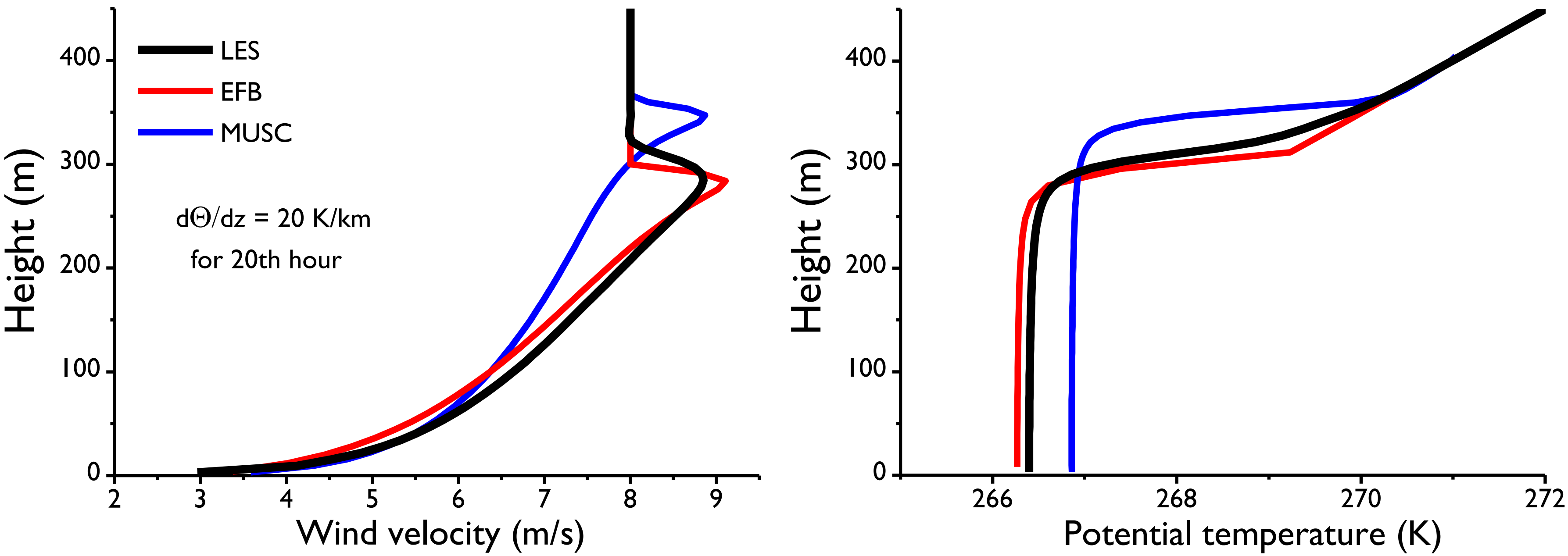




EFB is much closer to LES

Conventionally Neutral PBL: mean profiles

Same as GABLS1, but for zero surface heat flux: Initially homogeneous PBL evolves against very stable stratification in the free atmosphere causing the negative (downward) heat flux



Traditional theories overestimate PBL height and overwarms PBL

Concluding remarks

EFB closure shows good agreement with DNS and LES of:

- stably stratified Couette flows
- neutrally stratified PBL
- conventionally neutral PBL
- stably stratified GABLS1

Verification of EFB against DNS and LES shows obvious advantages of EFB compared to currently used closure models

DNS and LES for larger *z*/*L* are needed for further validation and inter-comparison

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Thank you for your attention

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